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ON SOME STEADY STATE TEMPERATURE FIELDS IN SEMI-INFINITE SOLIDS WITH CYLINDRICAL HEAT SOURCES

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An integral equation is obtained which describes the steady-state temperature field in a semi-infinite body with internal cylindrical heat sources (for cylinders of an arbitrary shape). For practical calculations a simplified equation is obtained by replacing the integral by a sum of a finite number of terms. The equations were used by the author to develop new methods of calculating the temperature in disc and drum rotors of heat turbines cooled by blowing-off the working fluid through distributed slots.

Consider an infinite plane with infinite rows of identical line sources and line sinks of constant density q. Let the distance between the rows be $2h_0$, let the spacing of the sources (sinks) in a row be S (Fig. 1), and let the temperatures of the sources and sinks be $+\infty$ and $-\infty$, respectively. Assume that the lines are the bases of open cylindrical surfaces, whose generating lines are parallel to each other. The equation of each line, $v = f(u)$, and its length l are given. The thermal conductivity of the solid λ is assumed to be constant. Let the isotherm which coincides with the line of symmetry between the rows of sources be t_B, let the x axis coincide with that isotherm, and let the y axis pass through a source-sink pair (Fig. 1).

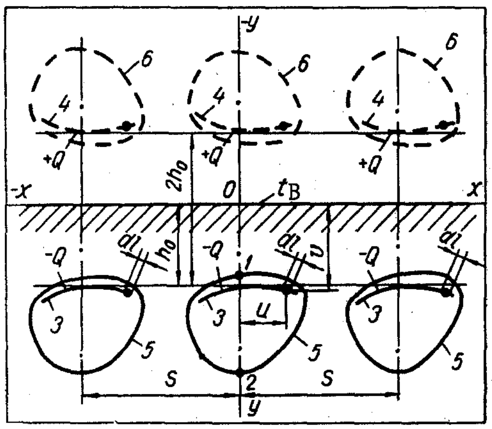

Fig. 1. Semi-infinite solid with internal cylindrical heat sources.

Under these conditions, the steady-state temperature distribution in the solid is given by the heat-conduction equation

$$\frac{\partial^2 t(x, y)}{\partial x^2} + \frac{\partial^2 t(x, y)}{\partial y^2} = 0. \tag{1}$$

Summing up, according to [1], the temperature fields due to line line elements dl (point sources and sinks) at an arbitrary point M (x, y) (y > 0), we obtain

$$d\Theta = d(t_B - t) = \frac{dQ}{4\pi\lambda} A,$$

$$dQ = q dl, \quad A = \ln \frac{\operatorname{ch}(2\pi/S)(y+v) - \cos(2\pi/S)(x-u)}{\operatorname{ch}(2\pi/S)(y-v) - \cos(2\pi/S)(x-u)}.$$

Integration along the length of the line yields

$$\Theta = t_B - t = \frac{q}{4\pi\lambda} \int_l A dl. \tag{2}$$

Equation (2) can be used to calculate the temperature field in a semi-infinite solid with internal cylindrical heat sources (or channels with given surface temperature), whose bases are identical isothermal contour lines, closed around the source lines (Fig. 1). By changing the shape of the source line, it is possible to obtain arbitrary channel profiles.

Consider the cooling of a semi-infinite solid by a system of such channels (the region y > 0 in Fig. 1). Let there be given the surface temperature t_B of the solid and two points (1 and 2) on the t_0 isotherm (boundary conditions of the first kind). Assume that $t_B > t_0$ (i.e., assume the existence of sources). In order to assure that the t_0 isotherm is a closed curve and encloses the sink line, let the points 1 and 2 be given on a vertical straight line (e.g., the y axis) above and below the sink line, respectively.

Rewriting equation (2) in relative units, we obtain

$$\bar{t} = \frac{t - t_0}{t_B - t_0} = 1 - \frac{1}{2\bar{R}l} \int_l A dl, \tag{3}$$

where $\overline{R} = 2\pi\lambda(t_B - t_0)/Q$, $Q = ql$.

Solution (3) satisfies equation (1) and the boundary condition at the boundary of the solid: at $y = 0$ $\overline{t} = 1$ or $t = t_B$.

The position of the sink line h_0 and its total strength Q are determined from the solution of the system of two equations obtained by applying the condition $\overline{t} = 0$ to the given points 1 and 2.

Consider the case when the line sink is a segment of a horizontal straight line with length l. Assuming that the y axis passes through the center of the segment (Fig. 2), we obtain

$$\overline{t} = 1 - \frac{1}{2\overline{R}l}\int_{-l/2}^{+l/2} \ln \frac{\operatorname{ch}(2\pi/S)(y + h_0) - \cos(2\pi/S)(x - u)}{\operatorname{ch}(2\pi/S)(y - h_0) - \cos(2\pi/S)(x - u)}\, du. \tag{3a}$$

Integrals of type (2), (3a) cannot be evaluated by elementary formulas. For practical calculations it is convenient to express these in finite-difference form according to the trapezoidal formula. The system of equations for h_0 and $\overline{R}(Q)$ can then be solved by the method of successive approximations.

Such an approximate solution was calculated for the case $(y_1 + y_2)/S = 0.6$, $(y_2 - y_1)/S = 0.4$, $l/S = 0.5$ and $n = 4$. The values of h_0 and $\overline{R}$ were found from this solution, and the t_0 isotherm was calculated point by point (Fig. 2). It can be seen that the t_0 isotherm has two discontinuities, which is due to the fact that equation (3a) has been expressed in finite-difference form in which segments of the line sink were replaced by point sinks.

The dash line represents a possible contour of the isotherm obtained from an exact solution. Its shape is close to a triangular channel. When the points 1 and 2 approach each other, the contour of the isotherm approaches the shape of a narrow plane slot. Therefore equation (3a) can be used for the approximate solution of the problem of cooling (heating) of a solid by a system of plane-slot channels.

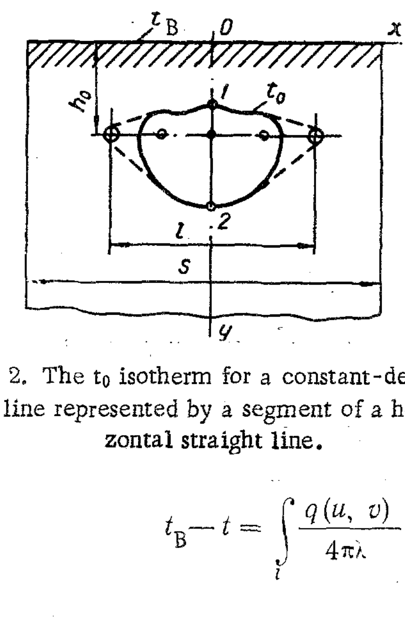

Fig. 2. The t_0 isotherm for a constant-density sink line represented by a segment of a horizontal straight line.

Equations, analogous to (3a), can be written down for sink segments represented by the equation of a circle, ellipse, parabola, hyperbola, etc. By specifying in this manner the shape of a sink line of constant density, one can obtain many difference channel contours. Thus, equation (3) can be used to solve a broad class of problems.

However, in all these cases the equations of the cooling contour are determined by the given shape of the sink line and can be quite different from the required cooling channel profile. In the general case, in order to make the contour of the t_0 isotherm coincide with a given profile, it is necessary to choose appropriately not only the shape of the sink line, but also the density distribution along this line. Thus, for channels with arbitrarly specified profile we can write

$$t_B - t = \int_l \frac{q(u,\ v)}{4\pi\lambda} \times \ln\frac{\operatorname{ch}(2\pi/S)\,[y + v(u)] - \cos(2\pi/S)(x - u)}{\operatorname{ch}(2\pi/S)\,[y - v(u)] - \cos(2\pi/S)(x - u)}\,dl. \tag{4}$$

Clearly, the functions v(u) and q(u, v) are determined, in accordance with the condition $t = t_0$, by the whole contour. This requires the solution of a system of integral equations with unknown limits of integration, which leads to considerable difficulties.

However, for practical calculations it is sufficient to express the generalized equation (4) in finite-difference form, by replacing the continuous sink distribution by point sinks, namely

$$\overline{t} = \frac{t - t_0}{t_B - t_0} = 1 - \sum_{i=1}^{m} \frac{1}{2\overline{R}_i} \ln \frac{\operatorname{ch}(y + h_{0i})\,2\pi/S - \cos(x - x_{0i})\,2\pi/S}{\operatorname{ch}(y - h_{0i})\,2\pi/S - \cos(x - x_{0i})\,2\pi/S}, \tag{5}$$

where $\overline{R}_i = 2\pi\lambda(t_r - t_0)/Q_i$.

In a more general case there can be several rows of channels, with different profiles and different spacing. In that case additional sums would appear in equation (5).

In order to determine the sink parameters (x_{0i}, h_{0i}, $\overline{R}_i$) one must set up a system of 3m equations obtained by applying the condition $\overline{t} = 0$ ($t = t_0$) at 3m points of the given profile. By specifying the number of sinks one can use an arbitrary number of points on the given profile and can solve the problem with arbitrary accuracy. During the solution of the problem one should remember that the heat sinks should lie inside the region enclosed by the given profile.

In most practical problems the solution can be simplified. Thus, for example, in the case of profiles symmetric with respect to the y axis the number of unknowns is halved. The number of unknowns can also be reduced by specifying the abscissas of some sinks. In the case of circular channels, when the radii are considerable smaller than h_i and S, one can assume $x_{0i} = x_i$, $h_{0i} = h_i$ (the sink coincides with center of the circle), and in that case the problem is reduced to the solution of a system of linear algebraic equations with unknown $z_i = 1/\overline{R}_i$.

Using equation (5), one can easily obtain an approximate solution for circular channels of equal depth and uniform spacing, assuming $m = 1$. The parameters h_0 and $\overline{R}$ are in this case determined from the relations

$$\frac{2\pi}{S} h_0 = \operatorname{arch}\left(\operatorname{ch}\frac{2\pi}{S} h \Big/ \operatorname{ch}\frac{2\pi}{S} r\right); \quad \overline{R} = \operatorname{arch}\left(\operatorname{sh}\frac{2\pi}{S} h \Big/ \operatorname{sh}\frac{2\pi}{S} r\right). \tag{6}$$

This solution is satisfactory when $r/S \leq 0.1$.

In another article [2] the author has shown some important characteristics of the temperature field described by (5), which were used to derive equations for the calculation of temperature in plane walls with internal cylindrical heat sources.

In practice one often has to apply boundary conditions of the third kind, when the temperatures and heat-transfer coefficients of the media adjoining the surfaces of the solid and flowing inside the channels are given. For the approximate solution of such problems it is recommended to use the method of an additional wall, the essence of which is the replacement of external thermal resistances by internal resistances. This reduces the problem to a problem with boundary conditions of the first kind. Specific examples of the application of this method are given in [2, 3, 4].

The temperature-field equations obtained here were used by the author to develop methods for calculating the temperature in disc and drum rotors of heat turbines cooled by injecting* the working fluid through distributed slots. The results of calculations based on this method were in good agreement with experimental data obtained on a thermal model of the rotor and by electric-analog methods [3, 4].

NOTATION

Q and $\overline{R}$ – sink strength and dimensionless thermal resistance, respectively, of unit length of a channel whose contour coincides with the isotherm t_0 ($\overline{t} = 0$); n – even number of equal intervals, into which a sink line is divided; $q(u, v)$ – sink density distribution along the line sink; $v = v(u)$ – equation of the sink line; m, x_{0i}, h_{0i} – number and coordinates of point sinks in a contour; $\overline{R}_i$ – dimensionless thermal resistance of point sinks, referred to the isotherms t_B and t_0; Q_i – sink strength; +Q and –Q – strengths of line sources and sinks, respectively; h – depth of centers of a row of circular channels; r – radius of circular channel; 3, 4 – open cylindrical heat sources with surface temperatures $-\infty$ and $+\infty$, respectively; 5, 6 – closed cylindrical heat sources with surface temperatures t_0 and t_0' ($t_0' > t_B > t_0$), respectively.

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*Studies of this problem indicate that this cooling method makes it possible to construct modern high-capacity, high-temperature steam and gas turbines mainly from well-known and relatively cheap perlitic steels.